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Plasma-wakefield acceleration of a positron beam

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Plasma-wakefield excitation by positron beams is examined in a regime for which the plasma dynamics are highly nonlinear. Three dimensional particle-in-cell simulations and physical models are presented. In the nonlinear wake regime known as the blowout regime for electrons, positron wakes exhibit an analogous ‘‘suck-in’’ behavior. Although analogous, the two wakefield cases are quite different in terms of their amplitudes, wavelengths, waveforms, transverse profiles, and plasma density dependence. In a homogenous plasma, nonlinear positron wakes are smaller than those of the corresponding electron case. However, hollow channels are shown to enhance the amplitude of the positron wakes.

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Significant advanced accelerator research is aimed at developing new technologies to reduce the size and cost of future elementary particle physics experiments based on colliding high-energy beams. One possible scenario for a future electron-positron collider is to use short plasma sections just before the collision point of an existing collider to boost the energy of each beam. In this case, a bunch from each beam line excites a plasma wake that is used to accelerate a trailing bunch from its respective beam line, thus wake excitation from both electron and positron bunches needs to be understood. Currently an experiment is underway at the Stanford Linear Accelerator Center (SLAC) to demonstrate such wakefield excitation by an electron bunch [1]. Simulations of this electron driven case exhibit a promising scaling of the maximum wakefield amplitude (E_o) with the bunch length (σ_z): nominally $E_o \propto 1/\sigma_z^2$ [2]. However, this scaling leads one inevitably into a nonlinear regime, where plasma electrons are blown out radially. It is not clear whether these results apply equally to positron cases for which plasma electrons are pulled inward. Moreover, there is a lack of experimental data for positron wakes.

This Rapid Communication describes three-dimensional (3D) particle-in-cell simulations and physical models for plasma-wakefield excitation by positron beams in a regime for which the plasma dynamics are highly nonlinear. For electrons this is often referred to as the blowout regime. The corresponding positron wakes are in an analogous ‘‘suck-in’’ regime. Although analogous, the two wakefield cases are quite different in terms of their amplitudes, wavelengths, waveforms, transverse profiles, and plasma density dependence.

We begin by summarizing the results of linear wakefield theory. Linear-wakefield theory has been validated by detailed computer simulations and several experiments [3–9] and applies equally to electrons and positrons. The linear response of a plasma to a relativistic bunch with bi-Gaussian density distribution is given by integrating the Green’s function for a single electron [7]. The result can be expressed approximately by

$$eE = \sqrt{n_o}(\text{eV/cm})\,\frac{n_b}{n_o}\,\frac{\sqrt{2\pi}k_p\sigma e^{-k_p^2\sigma_z^2/2}}{1+\dfrac{1}{k_p^2\sigma_r^2}}\sin k_p(z-ct), \tag{1}$$

where E is the wake electric field on the axis behind the bunch, n_o is the background plasma density in cm^{-3}, $k_p = \omega_p/c$ is the plasma wave number, $n_b \approx N/(2\pi)^{3/2}\sigma_z\sigma_r^2$ is the peak bunch density, N is the number of particles in the bunch, and σ_z and σ_r are the rms dimensions of the bunch. Equation (1) predicts the maximum wake amplitude to occur for a plasma density such that $k_p\sigma_z \cong \sqrt{2}$ for narrow bunches ($k_p\sigma_r \ll 1$) of fixed number of particles. For this matched plasma, the wakefield amplitude can be expressed as

$$eE_o \approx 240\,\text{MeV/m}\left(\frac{N}{4\times 10^{10}}\right)\left(\frac{0.6\,\text{mm}}{\sigma_z}\right)^2. \tag{2}$$

From this we see that the maximum wake amplitude for a given amount of charge scales as $1/\sigma_z^2$. However, this is strictly valid only within the limits of the linear theory; i.e., $n_b \ll n_o$ and $eE/m\omega_p c \ll 1$.

For typical parameters of interest for high gradient acceleration, these limits of the linear theory are exceeded. For example, in the recent wakefield experiment at SLAC [1], n_b is greater than n_o and $eE/m\omega_p c$ is on the order of 1. Thus, we turn instead to fully nonlinear particle-in-cell numerical simulations.

To study the scaling laws in this nonlinear regime of $n_b > n_o$ and narrow bunches, we employ the multidimensional (2D and 3D) fully self-consistent, and object-oriented model OSIRIS. This simulation model has been described in more detail elsewhere [10,11]. In a typical simulation, we follow 10^6–10^8 particles on a two or three- dimensional moving mesh of 10^5 to 10^6 cells of size 0.05 c/ω_p. The beam and plasma particles move according to the Lorentz force and their self-consistent electromagnetic fields, found by solving Maxwell’s equations on staggered grids. This code and others have been used previously [2] to model 2D, cylindrically symmetric nonlinear wakefields excited by electron bunches with densities a few times the plasma density. Here we extend that work by considering positrons, and by using fully 3D simulations. The 3D simulations in Cartesian geometry are used to validate the 2D cylindrical models.

The simulation results in Figs. 1–3 show a comparison of the wake structures and energy gain for electron and positron bunches. Physical beam parameters available at SLAC are used in the simulations ($N \sim 2\times 10^{10}$, $\sigma_z = 0.4$ mm, σ_r

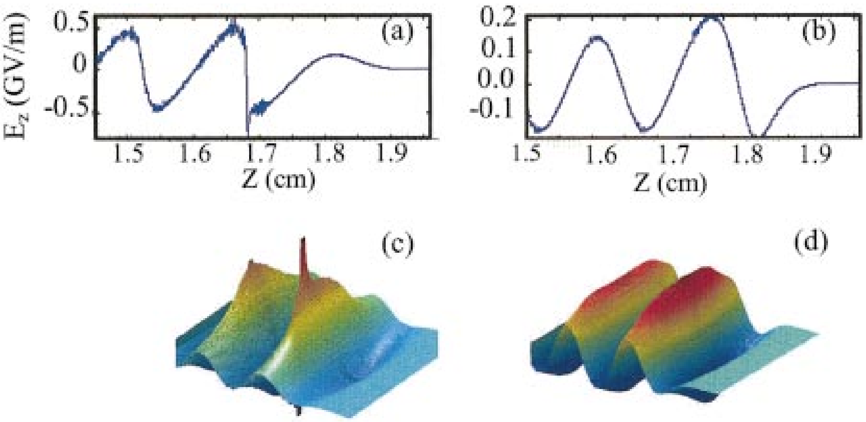

FIG. 1. (Color) Accelerating wake structures. E_z on axis for (a) electron and (b) positron bunches; surface plot of E_z vs x, z for (c) electron and (d) positron bunches. System size: 1.27 mm$\times$1.27 mm $\times$5.1 mm moving at c in the z direction; grid size: $dx=dy=12.7\ \mu$m, $dz=25.5\ \mu$m.

$=75\ \mu$m, normalized emittance $\gamma\varepsilon=15$ mm-mrad); and the plasma density ($4.3\times10^{14}\,\text{cm}^{-3}$) is chosen to satisfy the matching condition mentioned above ($\sigma_z\approx1.5c/\omega_p$). For this case, the ratio of n_b/n_o is 1.3, and as expected linear theory is not applicable. As is clear from the figures, there is an asymmetry between the electron and positron cases. In this regime, electron drivers cause the plasma electrons to blow out of the beam path, while positron drivers cause the plasma electrons to be sucked in to the axis (Fig. 4).

The longitudinal wake structure (Fig. 1) is less steepened for positrons than electrons and is broader in width. The peak-accelerating field by the positron bunch is about 210 MV/m, considerably lower than the electron wakefield of 830 MV/m and the linear theory [Eq. (1)] 250 MV/m. In addition, the peak decelerating field is about 170 MV/m, corresponding to a transformer ratio of 1.2. This is also considerably lower than for electron drivers.

In order to compare experimental observables, we plot in Fig. 2 the energy gain and loss of the electron and positron beams after propagating through 1.6 cm of plasma. The average energy gain (expressed in MeV/m) and the number of particles in 0.122 psec bins along the z axis are shown for each case. Interestingly, the disparity between bin-averaged energy gain of positrons and electrons (170 MeV/m versus 250 MeV/m) is much less than the disparity between the peak wakefields (Fig. 1). This is because the peak electron wake is very narrow and only a small fraction of electrons in a bin experience the full force. Note that to obtain an accelerated positron beam of small energy spread, a short trailing-beam could be loaded near the momentum peak (e.g., at $z=1.65$ cm in Fig. 2).

The transverse wakefields (Fig. 3) also differ considerably for positron and electron drivers. The focusing force on the positron driver is not constant in z, and is not as linear in r as in the electron case. Both of these "aberrations" can lead to undesirable emittance growth of a beam load. Fortunately, it is possible to mitigate these by detuning the plasma density for given beam parameters. For example, it is observed in simulations that a "sweet spot" that has constant focusing in z is formed in the acceleration region behind the positron driver when the bunch length to plasma wavelength ratio is raised (by 40% to $\sigma_z/\lambda_p\approx0.3$).

The simulations show new physical behavior with a positron bunch that is not predicted by linear theory. When the plasma density is mismatched slightly as described above, the accelerating wake peaks off the axis. The physical origin for this phenomenon is the phase mixing of radial plasma oscillations. The real space of plasma electrons (Fig. 4) shows electrons sucked in by the space charge of a positron bunch. However, electrons originating from different radial distances from the beam axis arrive at different times. This phase mixing reduces the size of the density compression and hence the wake as well. Furthermore, electrons that arrive at the axis first, then cross through the axis where they encounter electrons still rushing inwards from outer initial radii. This leads to density compressions that are peaked off axis.

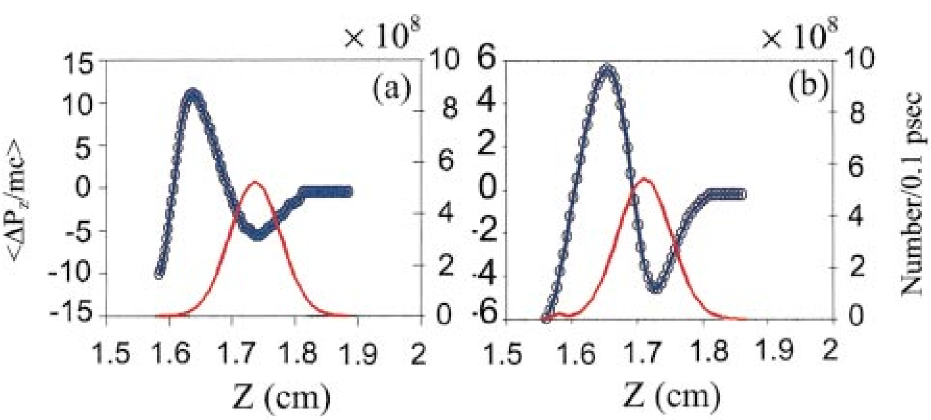

FIG. 2. (Color) Beam momentum gain (circles) vs z and beam current vs z for (a) electron and (b) positron beams after 1.59 cm propagation.

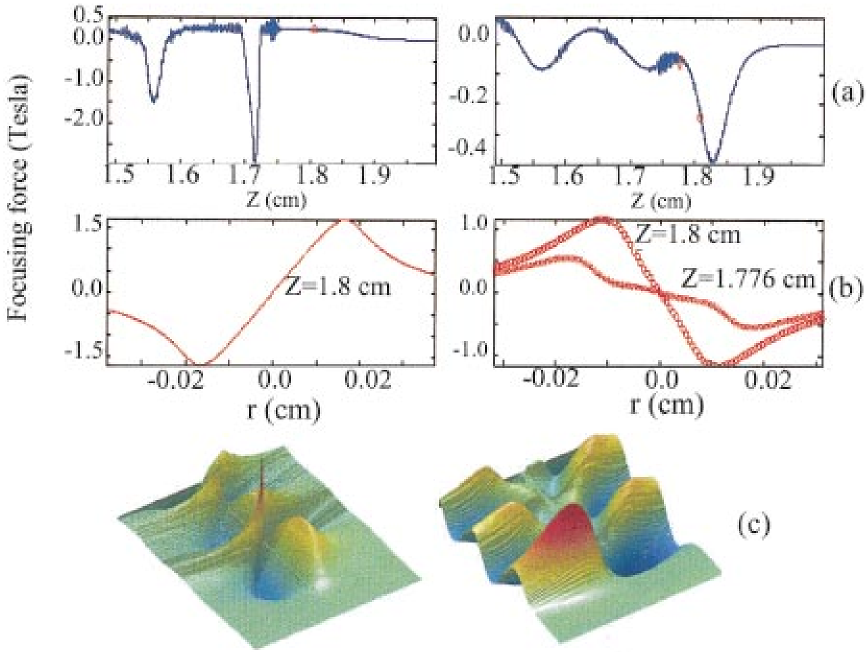

FIG. 3. (Color) Focusing wake ($E_r - B_\theta$) for electron (left) and positron (right) bunches: (a) slice at $r = 25\,\mu$m, (b) slice plots at z positions shown, and (c) surface plots.

The numerical expression for the suck-in time within a ring model approximation can be obtained. We consider each plasma electron to be a ring of radius r in 2D cylindrical geometry [12]. We assume the positron distribution to be bi-Gaussian. The motion of a ring with initial radius, r_o is given by

$$m\frac{d^2\xi}{dt^2} = 4\pi n_b e^2 r_b \frac{\sigma^2}{r_o - \xi}\left(1 - e^{-(r_o-\xi)^2/(2\sigma^2)}\right) e^{-(t-t_\sigma)^2 c^2/2\sigma_z^2} + 2\pi n_o e^2 (r_o - \xi) - 2\pi n_o e^2 r_o [r_o/(r_o-\xi)], \quad (3)$$

where $r = r_o - \xi$ and ξ is its inward displacement. The terms in Eq. (3) arise from the beam space charge, plasma ion space charge, and plasma electron space charge, respectively. Equation (3) can be numerically integrated to obtain exact solutions [11]. However, simple analytic expressions which illustrate the basic physics can be obtained by assuming that the beam radius is very small, $(n_b/n_o)\sigma_r^2/r_o^2 \gg 1$, and that the temporal profile is a step function. Then Eq. (3) can be approximated as

$$m(d^2r/dt^2) = -(2\pi n_b e^2 r_b^2)/r. \quad (4)$$

We solve this differential equation for the suck-in time, the time it takes a plasma electron to reach the axis from its initial radius r_o:

$$\tau_{\rm col} \approx \sqrt{\pi}(r_o/\omega_{pb} r_b), \quad (5)$$

where $\omega_b = \sqrt{4\pi n_b e^2/m}$ is the beam plasma frequency. The electrons near the axis arrive more quickly than those farther away; thus, there is no time for which there is a well-defined density compression. For electrons originating farther away from the axis the expression is more complicated, but Eq. (5) clearly illustrates the problem of phase mixing.

It is interesting that in the suck-in regime, the time scale for the collapse in Eq. (5) for the inner core of electrons is independent of the plasma density. However, it does depend on the beam plasma frequency ω_{pb}, which is a function of the beam radius. This implies that the beam radius cannot be allowed to oscillate (i.e., betatron oscillations) if the phase of the acceleration peak is not to slip. As discussed in Ref. [2],

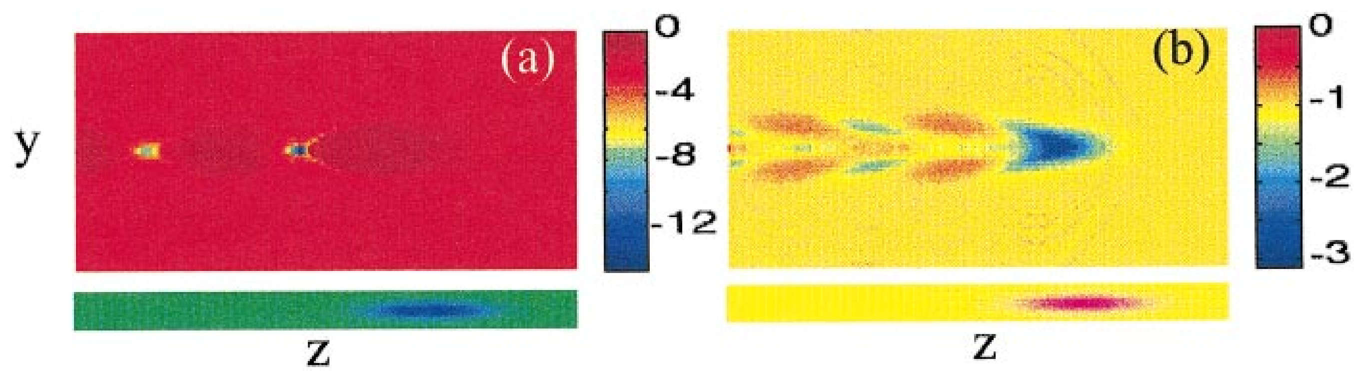

FIG. 4. (Color) Real space of plasma electrons in (a) electron wake and (b) positron wakes. Real space of beam is shown below.

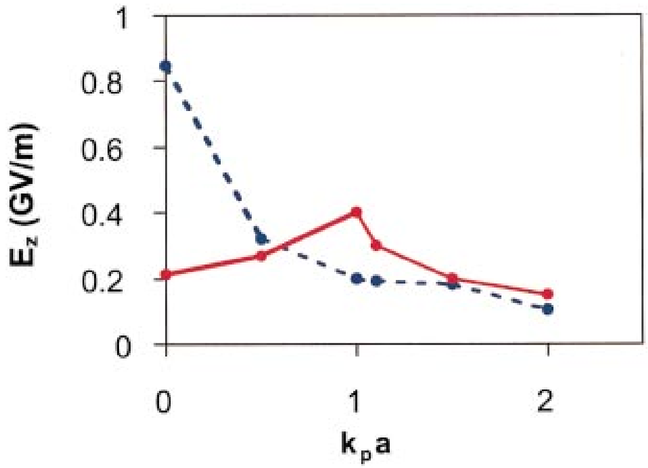

FIG. 5. (Color) Peak wake amplitude vs radius of the channel for electron (dashed) and positron beams.

transverse emittance matching can be used to minimize if not eliminate betatron oscillations.

As stated above, the primary limit to positron wake amplitude is due to the phase mixing of plasma electrons coming from different radii. One solution is to use a plasma in the shape of a hollow cylinder [13]. Such a plasma can be made easily by uv ionization of a long gas column by a laser for which the center is blocked [14].

To test this concept we have simulated positron and electron wakes in hollow plasma channels. The results in Fig. 5 show that the amplitude of the positron wake increases as the channel radius increases toward c/ω_p. Interestingly, the nonlinear wake profiles for positrons and electrons reverse roles in the hollow and homogeneous plasmas. The wake of the positron driver becomes spiked on the axis while the electron wake in a hollow plasma channel becomes uniform in r.

In summary, we have shown with 3D simulations that the nonlinear wake of a positron bunch is smaller than that of an electron bunch in homogenous plasma, but it can be made comparable to the electron wake by employing a hollow plasma. We comment that simulations of positron wakes in 2D cylindrical geometry raise code issues concerning the proper handling of particles near the axis. Despite the concern that the singularity at the axis can cause significant errors in the charge deposition, we found that the 2D cylindrical results were nearly identical to the 3D results with the same cell sizes. To assess the feasibility of positron wakes for future collider applications, further work directed toward the beam loading of positrons and their long term propagation are needed.

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